Geometry of large genus flat surfaces

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Square-tiled surface

Definition

A ***square-tiled surface*** *(or* ***origami****) is an oriented compact connected surface obtained by gluing a finite number of isometric squares along parallel sides by translation (right $\leftrightarrow$ left, up $\leftrightarrow$ down).*

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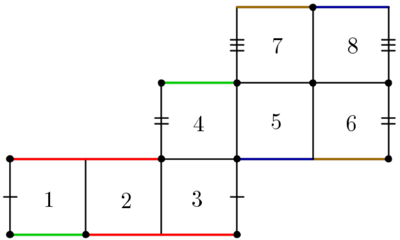
Labelled origami: squares are numbered



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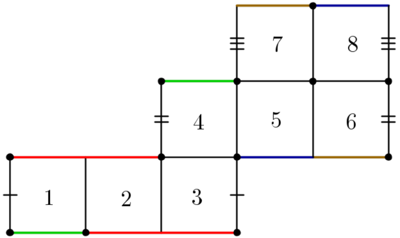


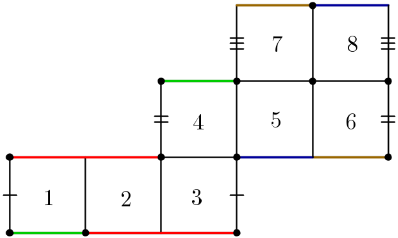
$h = (1,2,3)(4,5,6)(7,8)$
$v = (1,2,3,4)(5,7,6,8)$

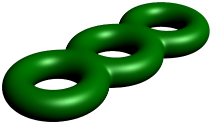
Equivalent definition

A labelled origami with N squares is a pair of permutations $(h, v) \in S_N \times S_N$ acting transitively on $\{1, \dots, N\}$.

Geometry of square-tiled surfaces

- topology (genus)





Geometry of square-tiled surfaces

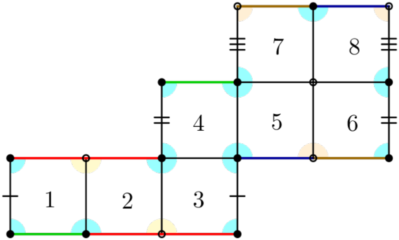
- topology (genus)
- flat metric with conical singularities (coming from the euclidean metric on $\mathbb{R}^2$)

Degre k_i of a singularity: number of extra turns.

Euler-Poincaré

$$2g - 2 = \sum_i k_i.$$

$k_1 + 1, \ldots, k_n + 1$ is the cycle type of $v^{-1}h^{-1}vh$.



$g = 3,\ k = 4$
$v^{-1}h^{-1}vh = (2, 7, 3, 4, 6)$

Geometry of square-tiled surfaces

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- area: number of squares

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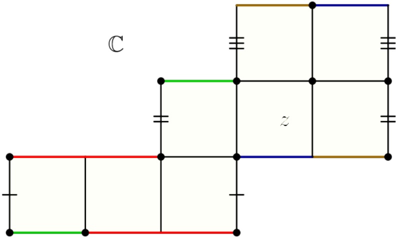

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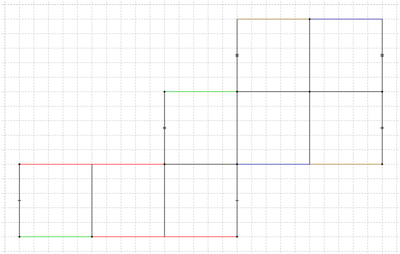
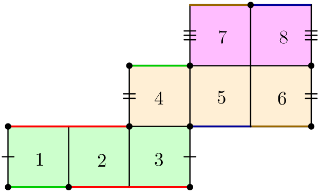


Cylinders

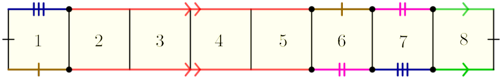
Definition

*A **cylinder** is a maximal collection of parallel closed geodesics*

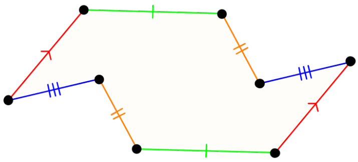
- **3 cylinders** SQT with 8 squares, genus 3, one singularity of degree 4



- **1 cylinder** SQT with 8 squares, genus 3, one singularity of degree 4.



Translation surfaces



Flat metric
Conical angles $(d+1)\cdot 2\pi$

$\updownarrow$

Riemann surface
with a holomorphic 1-form
(Abelian differential)
zeros of degree d

Gauss-Bonnet / Euler-Poincaré:

$$\sum_i d_i = 2g - 2$$

Pair of transverse foliations (horizontal and vertical)

Moduli space of translation surfaces

$$\mathcal{H}_g = \{\text{translation surfaces of genus } g\}/\text{cut and paste} = \bigsqcup_{\underline{d} \vdash 2g-2} \mathcal{H}(\underline{d})$$

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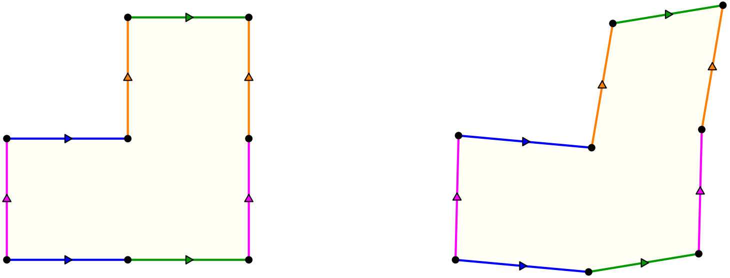

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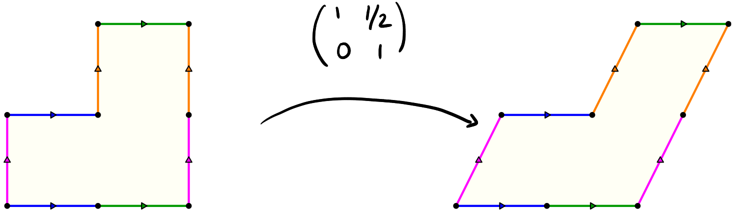

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• Lebesgue measure in local coordinates
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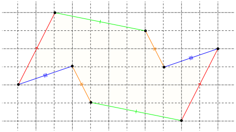

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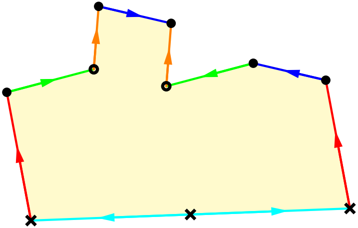
• Square-tiled surfaces are "integer" points in the stratum

$$\left|\{\text{SQT of type}(k_1, \ldots, k_n) \text{ with } \leq N \text{ squares}\}\right| \sim cN^d \text{ as } N \rightarrow \infty$$

$c = Vol\mathcal{H}(k_1, \ldots, k_n)$ is the Masur-Veech volume of $\mathcal{H}(k_1, \ldots, k_n)$.

Half-translation surfaces ans SQTs: strata $\mathcal{Q}(\underline{k})$

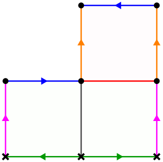


Flat metric
Conical angles $(k+2)\cdot\pi$

$\updownarrow$

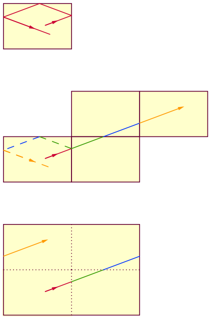
Riemann surface
with a quadratic differential
(at most simple poles)
singularities of order $k \geq -1$

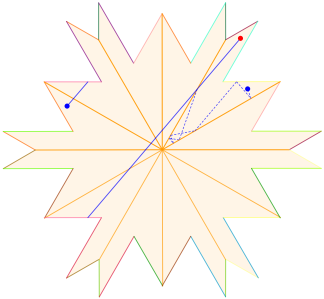
Example of a SQT in the stratum $\mathcal{Q}(2,-1,-1)$ (genus 1).
2 cylinders

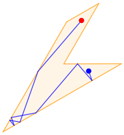


Why do we care about (half-)translation surfaces and their moduli spaces?

Motivation: rational polygonal billiards







Why do we care about (half-)translation surfaces and their moduli spaces?

Dynamical behaviour on individual surfaces $\leftrightsquigarrow$ $SL(2,\mathbb{R})$-orbit closure

Theorem (Lelièvre-Monteil-Weiss, 2016)

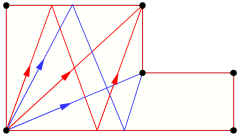
For any rational billiard P, for any $x \in P$, there are at most finitely many points y for which there is no billiard trajectory between x and y.

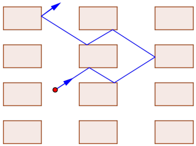
Theorem (Eskin-Mirzakhani, E-M-Mohammadi 2015, 2018)

Description and structure of the $SL(2,\mathbb{R})$-orbit closures in the moduli space and classification of the $SL(2,\mathbb{R})$-invariant measures.

Why do we care specifically about square-tiled surfaces?

Counting square-tiled surfaces provide estimations for the volumes of the moduli spaces and other quantitative invariants.





Theorem (Athreya-Eskin-Zorich, 2012)

As $L \to \infty$ the number of trajectories in the red family is $\frac{1}{2\pi}\frac{L^2}{area}$.
As $L \to \infty$ there are 4 times more trajectories in the blue family.

Theorem (Delecroix-Hubert-Lelièvre, 2011)

The diffusion rate for the wind-tree model is 2/3:

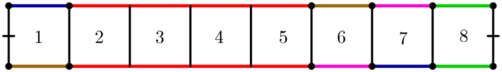
$$diam(\text{traj. at } t) \sim t^{2/3}.$$

Equidistribution of SQTs and uncorrelation results

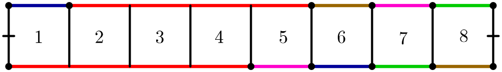
Square-tiled surfaces of type $(k_1, \ldots, k_n)$ with "fixed combinatorics"

- number of horizontal cylinders
- number of horizontal and vertical cylinders
- how cylinders are glued together
- topological type of cylinders (e.g. separating/non separating)

Example of a SQT in $\mathcal{H}(5)$ with 1 hor. cyl. (3 vert.cyl.):



Example of a SQT in $\mathcal{H}(5)$ with 1 hor. cyl. and 1 vert. cyl.:



Equidistribution of SQTs and uncorrelation results

Theorem

"The SQTs with fixed combinatorics equidistribute in the stratum". E.g:

$$\lim_{N\to\infty} \frac{\left|\{\textbf{1-cyl}\ SQT\ of\ type\ \underline{k}\ with\ \leq N\ squares\}\right|}{N^d} = cyl_1 > 0$$

$$\lim_{N\to\infty} \frac{\left|\{\textbf{1-cyl}\ SQT\ of\ type\ \underline{k}\ \textbf{with 1 vert. cyl.}\ and \leq N\ sq.\}\right|}{N^d} = cyl_{1,1} > 0$$

where d is the dimension of the ambiant stratum.

Theorem

"Hor. combinatorics and vert. combinatorics are asympt. uncorrelated." E.g.

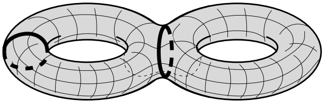
$$\frac{cyl_{1,1}}{cyl_1} = \frac{cyl_1}{Vol}.$$

Interlude on multicurves

Fix S a smooth oriented closed surface of genus $g \geq 2$.

Interlude on multicurves

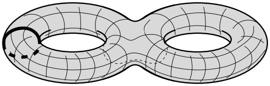
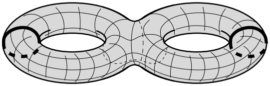
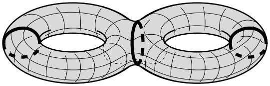
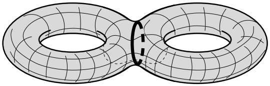
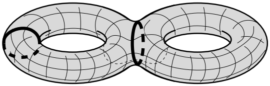
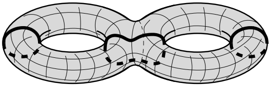
Fix S a smooth oriented closed surface of genus $g \geq 2$.
A *multicurve* on S is a formal sum $\gamma = \sum_{i=1}^{k} m_i \gamma_i$ with $m_i \in \mathbb{Z}_+$ and γ_i are non-contractible simple closed curves on S pairwise non-isotopic (up to free homotopy).



If all $m_i = 1$ we say that γ is *reduced*.

Topological types of reduced multicurves for $g = 2$

Topological type of a multicurve: MCG orbit: topology of the pieces (genus, number of boundaries) after cutting along the curves, and the way they are glued together.

1 comp	2 comp	3 comp
		
		

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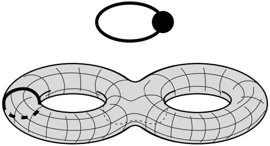	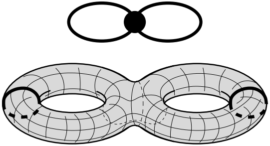	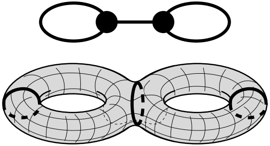
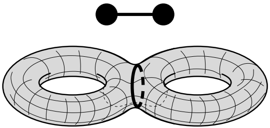	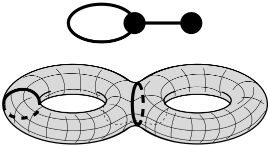	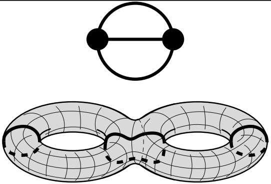

The topological type of a multicurve is encoded by a ”stable” graph

Result of Mirzakhani on the count of multicurves

Theorem (Mirzakhani '08)

For any multicurve γ_0 and any hyperbolic surface X of genus g

$$\mathrm{Card}\{\gamma : \textit{top. type of } \gamma \textit{ is } [\gamma_0] \textit{ and } \ell(\gamma) \leq L\} \sim B(X) \cdot \frac{c(\gamma_0)}{b_g} \cdot L^{6g-6},$$

as $L \to +\infty$, where $b_g := \int_{\mathcal{M}_g} B(X) dX = \sum_{\textit{all } [\gamma_0]} c(\gamma_0)$

Example: frequencies $\frac{c(\gamma_0)}{b_g}$ for $g = 2$

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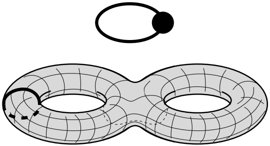
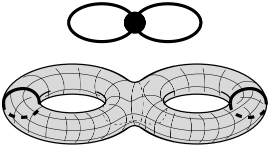
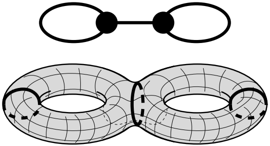
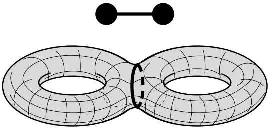
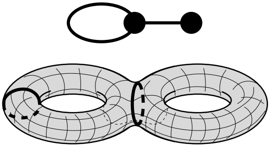
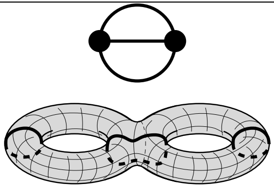
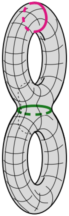
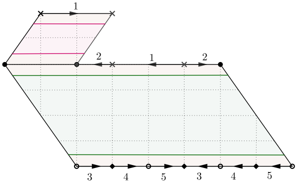
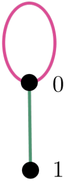
 25%	 53%	 11%
 0.5%	 2%	 7.4%

Combinatorics of SQT and multicurves on surfaces

For a (half-translation) square-tiled surface, the core curves of each cylinder form a reduced multicurve on the surface.

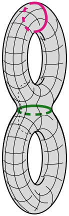


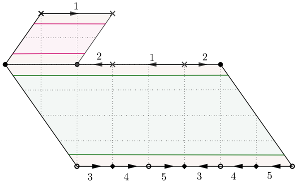


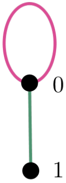


Combinatorics of SQT and multicurves on surfaces

For a (half-translation) square-tiled surface, the core curves of each cylinder form a reduced multicurve on the surface.





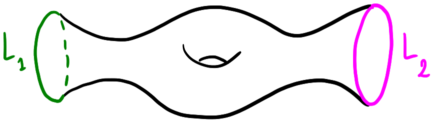


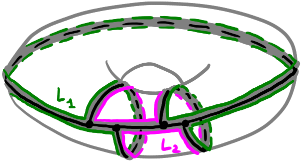
Fact: The frequency $c(\gamma_0)/b_g$ of multicurves of type γ_0 and the frequency c/Vol of SQTs of corresponding topological type **coincide**!

Examples: 1-component multicurves/ 1-cylinder SQTs, Separating curves/separating cylinders, etc.

Why frequencies are the same?

Hyperbolic surface with boundaries VS Ribbon graph





$$\sim \sum_{\alpha \vdash 3g-3+n} \frac{\int_{\overline{\mathcal{M}}_{g,n}} \psi_1^{\alpha_1} \dots \psi_n^{\alpha_n}}{\alpha_1! \dots \alpha_n!} L_1^{2\alpha_1} \dots L_n^{2\alpha_n} \text{ as } L_i \to \infty \text{ (Mirzakhani, Kontsevich)}$$

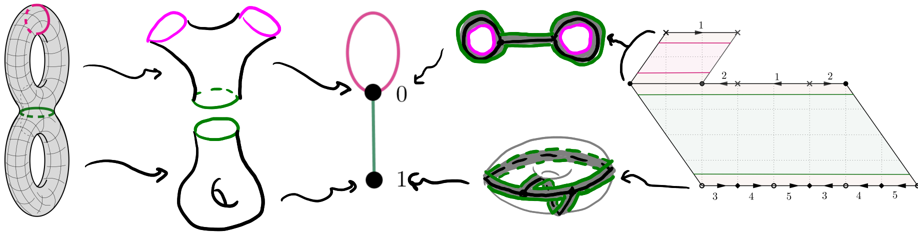
Facts (Bowditch-Epstein, Mondello, Do, . . .):

- Moduli spaces are homeomorphic
- The Kontsevich volume form is a limit of Weil-Petersson volume forms (after some renormalization)
- Hyperbolic surfaces with large boundaries ”resemble” ribbon graphs.

Why frequencies are the same?

Cut hyperbolic surfaces along geodesics:

Cut (half-translation) SQTs along cylinders:



The pieces are glued together along the same ”stable” graph (topological type of the multicurve / the decomposition into cylinders).

Large genus asymptotics: half-translation case

Here we assume that the half-translation surfaces have no singularities of angle π / the hyperbolic surface has no cusps.

- Separating 1-cylinder SQTs / simple closed curves

Theorem

$$\frac{c(sep)}{c(nonsep)} \sim \sqrt{\frac{2}{3\pi g}} \cdot \frac{1}{4^g} \quad as\ g \to \infty.$$

- Proportion of 1-cylinder surfaces / 1-component multicurves

Theorem

$$\frac{cyl_1}{\mathrm{Vol}} \sim \sqrt{\frac{\pi}{24g}} \quad as\ g \to \infty.$$

Large genus asymptotics: half-translation case

- Distribution of number of cylinders / number of components:

Theorem

It converges in a strong sense to the Poisson distribution of parameter $\lambda_g = \log(6g-6)/2$.

Same convergence as the the number of cycles of random permutations to $\mathrm{Poi}_{\log(n)}$ [Hwang,Nikeghbali-Zeindler].

Large genus asymptotics: half-translation case

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- Global separation:

Theorem

All singularities of a SQT are located on the same horizontal layer with probability that tends to 1 when g tends to infinity. A reduced multicurve does not separate the surface with probability that tends to 1 when g tends to infinity.

Large genus asymptotics: translation case

- Proportion of 1-cylinder surfaces

Theorem

$$\frac{cyl_1}{\mathrm{Vol}} \sim \frac{1}{4g} \quad as\ g \to \infty.$$

It holds for SQTs of fixed type $\underline{k}$ (stratum $\mathcal{H}(\underline{k})$).

Large genus asymptotics: translation case

- Proportion of 1-cylinder surfaces

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- Distribution of number of cylinders

Conjecture

It converges to the distribution of the number of cycles of random uniform permutations of S_{2g+n-1} (uniformly on the type $\underline{k} \dashv 2g-2$).

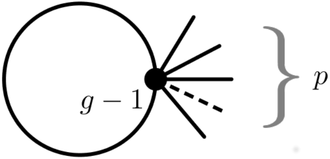
Outline of the proof: recent advances

Vol($\mathcal{Q}_{g,p}$) and Vol($\mathcal{H}_g$)

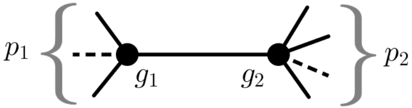
- Eskin-Okounkov $\sim$ '00, '05: algorithms for small dimension
- Athreya-Eskin-Zorich '12: closed formulas for Vol $\mathcal{Q}_{0,p}$
- Chen-Möller-Zagier '18 Vol($\mathcal{H}_g$) as $g \to \infty$ (gen. Aggarwal '19)
- DGZZ '18, Vol($\mathcal{Q}_{g,p}$) as a sum over stable graphs
- Chen-Möller-Sauvaget-Zagier '19: Vol($\mathcal{H}_g$) as Hodge integrals
- Andersen-Borot-Charbonnier-Delecroix-Giacchetto-Lewanski-Wheeler '19: topological recursion for Vol($\mathcal{Q}_{g,p}$) (from DGZZ '18)
- Chen-Möller-Sauvaget '19 Vol($\mathcal{Q}_{g,p}$) as Hodge integrals, and $p \to \infty$
- Aggarwal '19 Vol($\mathcal{Q}_{g,p}$) as $g \to \infty$ based on [DGZZ '18].
- Kazarian '19 , Yang-Zagier-Zhang '20: quadratic recursion for Vol($\mathcal{Q}_{g,p}$) based on [CMS].

$cyl_1(\mathcal{Q}_{g,p})$ and $cyl_1(\mathcal{H}_g)$

- DGZZ Explicit formula for $cyl_1(\mathcal{H}_g)$ (via characters of the symmetric group)
- DGZZ Explicit formula for $cyl_1(\mathcal{Q}_{g,p})$ as a sum over stable graphs:



$$16 c_{g,p} \sum_{k=0}^{3g-4} \langle \tau_k \tau_{3g-4-k} \rangle \sum_{i=0}^{p} \binom{p}{i} \binom{3g-4+p}{i+k}$$
$$\underset{p\to\infty}{\sim} \left(\frac{2^{4g-3}}{\sqrt{\pi}} \sum_{k=0}^{3g-4} \langle \tau_k \tau_{3g-4-k} \rangle \right) p^{g-\frac{1}{2}} 4^p$$
$$\underset{g\to\infty}{\sim} \left(16 \sqrt{\frac{2}{3\pi}} \left(\frac{16}{3}\right)^{p-4} \right) \frac{1}{\sqrt{g}} \left(\frac{8}{3}\right)^{4g}$$



$$\frac{c_{g,p}}{g! 24^g} \sum_{p_1=0}^{p} \binom{p}{p_1} \sum_{g_1=0}^{g} \binom{g}{g_1} \binom{3g-4+p}{3g_1-2+p_1}$$
$$\underset{p\to\infty}{\sim} \left(\frac{2^{2g-7}}{\sqrt{\pi} 3^g g!} \right) p^{g-\frac{1}{2}} 4^p$$

negligible as $g \to \infty$

$$p = cyl_{1,1}/cyl_1 = cyl_1/\,\mathrm{Vol}$$

- Proving the following asymptotics of $p_{g,p}$ or p_g by direct combinatorial arguments is still an open problem!

$$p(\mathcal{Q}_{g,p}) \underset{p\to\infty}{\sim} b_g p^{\frac{g-1}{2}} \left(\frac{8}{3}\right)^p$$

$$p(\mathcal{Q}_{g,p}) \underset{g\to\infty}{\sim} c\frac{1}{\sqrt{g}}$$

$$p(\mathcal{H}_g) \underset{g\to\infty}{\sim} \frac{1}{4g}$$

Results : distribution of the number of components

For a random variable X taking values in $\mathbb{Z}_+$,

$$\mathbb{E}(t^X) = \sum_{k=1}^{\infty} \mathbb{P}(X = k)t^k.$$

Example : Poisson distribution of parameter λ

$$\mathbb{P}(X = k) = \frac{\lambda^k e^{-\lambda}}{k!}, \quad \mathbb{E}(t^X) = e^{\lambda(t-1)}$$

For X and Y independent, $\mathbb{E}(t^{X+Y}) = \mathbb{E}(t^X)\mathbb{E}(t^Y)$.

Definition

X_n converges mod-Poisson with parameters λ_n and limiting function $G(t)$ if $\exists R > 1$, $\varepsilon_n \to 0$, $\forall t \in \mathbb{C}$ such that $|t| < R$,

$$\mathbb{E}(t^{X_n}) = e^{\lambda_n(t-1)} G(t) \big(1 + O(\varepsilon_n)\big)$$

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$$\mathbb{E}(t^{X_n}) = e^{\lambda_n(t-1)} G(t)\big(1 + O(\varepsilon_n)\big)$$

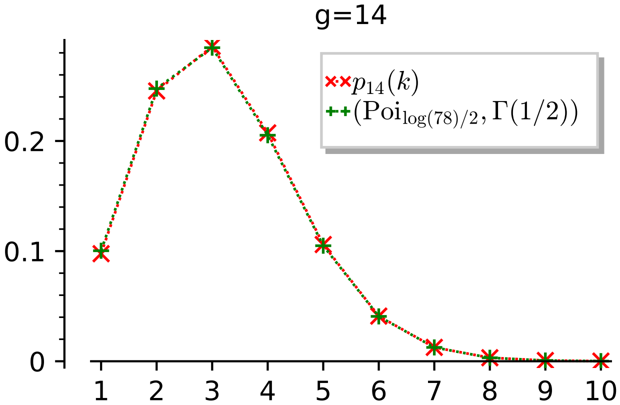
Theorem (Hwang '94, Nikeghbali-Zeindler '13)

The number of cycles in a uniformly random permutation of S_n converges mod-Poisson with parameter $\lambda_n = \log(n)$ and limiting function $G(t) = \frac{t}{\Gamma(1+t)}$. ($R = \infty$ and ε_n) $= 1/n$).

Theorem (DGZZ)

The number of cylinders in a random square-tiled surface of genus g (OR number of components of a multicurve on a genus g surface) converges mod-Poisson with parameter $\lambda_g = \log(6g-6)/2$ and limiting function $G(t) = t\,\Gamma(\frac{3}{2})/\Gamma(1+\frac{t}{2})$. ($R = 8/7$ and $\varepsilon_g = g^{-\delta(U)}$ on compacts U).

Results : distribution of the number of components



Exact distribution of number of components (coeffs of $\mathbb{E}(t^{K_g(\gamma)})$)
Mod-Poisson convergence (coeffs of $e^{\lambda_g(t-1)} \cdot G(t)$)